Reciprocity, Inequity Aversion, and Oligopolistic Competition,[†]

Luís Santos-Pinto

Universidade Nova de Lisboa, Departamento de Economia
Campus de Campolide, PT-1099-032, Lisboa, Portugal
Email address: lspinto@fe.unl.pt

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Abstract

This paper studies how reciprocity and inequity aversion influence the behavior of firms in imperfectly competitive markets. The paper shows that if reciprocal firms compete à la Cournot, then they are able to sustain "collusive" outcomes under a *positive reciprocity* equilibrium. By contrast, Stackelberg warfare outcomes may emerge under a *negative reciprocity* equilibrium. The results for inequity aversion are similar. Cournot competition between inequity averse firms can be harmful to consumers if it leads to equilibria where firms feel compassion toward each other. However, in equilibria where inequity averse firms are envious of each other consumers are better off than if firms were selfish. The paper also shows that only under very restrictive conditions does reciprocity or inequity aversion have an impact on Bertrand competition. Finally, the paper shows that non-selfish preferences have a greater impact on equilibrium outcomes in markets with a small number of firms.

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1 Introduction

Many experiments show that individuals are not only motivated by material self-interest, but also care about the well-being and the intentions of others. Some experiments show that many individuals are willing to incur losses to punish those who treat them unkindly or to reward those who treat them kindly. This type of behavior is called preferences for reciprocity. Other experiments find that many individuals are willing to give up some material payoff to move in the direction of more equitable distributions of payoffs. This type of behavior is called inequity aversion.[1]

Preferences for reciprocity and inequity aversion have been shown to explain behavior in bargaining games and in trust games.[2] For example, in ultimatum games offers are usually much more generous than predicted by equilibrium and low offers are often rejected. These offers are consistent with an equilibrium in which players make offers knowing that other players may reject allocations that appear unfair.[3]

The impact of non-selfish preferences on strategic interactions between firms has not received much attention.[4] The only exceptions are Bolton and Ockenfels (2000) and Santos-Pinto (2006). Bolton and Ockenfels (2000) find that of inequity aversion has no impact on Cournot and Bertrand competition.[5] Santos-Pinto (2006) shows that inequity aversion is able to organize most of the experimental evidence on endogenous timing games.[6] This happens because inequity aversion makes symmetric outcomes more attractive to players than asymmetric outcomes.

The paper starts by extending the Cournot model of quantity competition by incorporating preferences for reciprocity. We assume that a reciprocal firm cares about its own material payoff but also about the intentions of its rivals. If a reciprocal firm expects the output of its rivals to fall short of its own perception of their fair output, then the firm is willing to sacrifice some of its material payoff to increase its rivals' material payoffs. This assumption captures positive

[1] Some individuals also display altruism and others spitefulness. This paper does not study the impact of altruism and spitefulness on oligopolistic competition.

[2] Camerer (2003) and Sobel (2005) provide reviews of this literature.

[3] Economists have also began to study the implications of non-selfish preferences in optimal contracts. For example, Englmaier and Wambach (2002) study optimal contracts when the agent suffers from being better off or worse off than the principal. Biel (2003) studies how the optimal incentive contract in team production is affected when workers are averse to inequity. Sappington (2004) studies inequity aversion in adverse selection contexts.

[4] The impact of non-selfish preferences on perfectly competitive markets has been considered in several studies. Bolton and Ockenfels (2000) and Fehr and Schmidt (1999) show how the competitive prediction of the ultimatum game with many proposers and one responder studied by Prasnikar and Roth (1992) continues to hold under the assumption that some individuals in the population care about inequity aversion. Segal and Sobel (2004) show that interdependent preferences have no impact on equilibrium outcomes of perfectly competitive markets.

[5] We discuss their findings in Section 5.

[6] The prediction of asymmetric equilibria with Stackelberg outcomes is clearly the most frequent result in the endogenous timing literature. Several experiments have tried to validate this prediction empirically, but failed to find support for it. By contrast, the experiments find that simultaneous-move symmetric outcomes are modal.

reciprocity. By contrast, if a reciprocal firm expects its rivals to produce more than its own perception of their fair output, then the firm is willing to sacrifice some of its material payoff to reduce its rivals' material payoffs. This assumption captures negative reciprocity. Preferences of firms are assumed to be common knowledge.

The main result in the paper, Theorem 1, shows how reciprocal firms' perceptions of the fair output of their rivals change the outcome of Cournot competition. The result shows that if the weight of preferences for reciprocity in firms' payoff functions is not too excessive and the marginal returns from increasing production are decreasing with firms' perceptions of the fair output of their rivals, then the higher are firms' perceptions of the fair output of their rivals the lower will be the set of Cournot-Nash equilibria.

Next, the paper states three results that compare the equilibrium outcome of Cournot competition with selfish firms to that of Cournot competition with reciprocal firms. The first result, Proposition 1, shows that if reciprocal firms perceive the fair output of their rivals to be equal to the equilibrium output that the rivals would produce in the game with selfish firms, then they will produce the same quantities as selfish firms. In this case reciprocity shifts the firms' best replies (by comparison with the best replies of selfish firms) but does not change the equilibrium of the game.

The second result, Proposition 2, shows that if reciprocal firms compete à la Cournot, then they are able to sustain "collusive" outcomes under a *positive reciprocity* equilibrium.[7] By a positive reciprocity equilibrium we mean a Cournot-Nash equilibrium where firms feel positive reciprocity towards their rivals. A positive reciprocity equilibrium happens whenever reciprocal firms perceive the fair output of their rivals to be greater than the equilibrium output that the rivals would produce in a game with selfish firms. In a positive reciprocity equilibrium the rivals of a firm produce less than the firm's perception of the fair output of the rivals. In such an equilibrium a reciprocal firm produce less than a selfish firm in order to reward its rivals for producing less than its perception of the fair output level of its rivals. A positive reciprocity equilibrium is good for firms since it increases firms' material payoffs (by comparison with the material payoff of selfish firms) and provides firms' some payoff gains from positive reciprocity. A positive reciprocity equilibrium is bad for consumers since it leads to collusive outcomes.

Finally, Proposition 3, shows that Stackelberg warfare outcomes may emerge under a *negative reciprocity* equilibrium.[8] By a negative reciprocity equilibrium we mean a Cournot-Nash equilibrium where firms feel negative reciprocity towards their rivals. Such an equilibrium happens whenever reciprocal firms perceive the fair output of their rivals to be smaller than the equilibrium output that the rivals would produce in the game with selfish firms. In a negative reciprocity equilibrium the rivals of a firm produce more than the firm's perception

[7] Throughout the paper we consider that collusive outcomes describe situations where reciprocal firms produce less than the Cournot-Nash quantities of selfish firms.

[8] We consider that Stackelberg warfare describes a situation where reciprocal firms produce more than the Cournot-Nash outputs of selfish firms.

of the fair output of the rivals. In such an equilibrium a reciprocal firm produces more than a selfish firm in order to punish its rivals for producing more than its perception of the fair output level of its rivals. A negative reciprocity equilibrium is bad for firms since it reduces firms' material payoffs (by comparison with the material payoff of selfish firms) and makes firms incur payoff loses from negative reciprocity. A negative reciprocity equilibrium is good for consumers since it leads to Stackelberg warfare outcomes.

The paper proceeds by studying the impact of inequity aversion on Cournot competition. We assume that an inequity averse firm cares about its own monetary payoff and, in addition, would like to reduce the difference between its payoff and its rivals' payoffs. More specifically, such a firm is assumed to dislike advantageous inequity (it feels compassion) and also to dislike disadvantageous inequity (it feels envy).[9] The paper finds that the impact of inequity aversion on Cournot competition is similar to that of preferences for reciprocity.

We also state results that show that the set of Nash equilibria of Cournot competition when firms are averse to inequity changes monotonically with compassion and envy. If there is quantity competition and firms' degree of envy increases, then the largest Nash equilibria of the Cournot game moves closer to the perfectly competitive outcome.[10] By contrast, if there is quantity competition and compassion between firms increases, then the smallest Nash equilibria of the Cournot game moves closer to the collusive outcome.

Additionally we find that piecewise linear inequity aversion between firms can give rise to a continuum of symmetric equilibria. The paper also shows that as the number of firms grows the impact of piecewise inequity aversion on the set of Nash equilibria of a n-firm game vanishes. This happens because it takes only one selfish firm to destroy the continuum of equilibria generated by piecewise linear inequity aversion.

When there is price competition in homogeneous products, then piecewise linear inequity aversion between firms either has no impact on the set of equilibria or it can raise firms' prices. This happens because under Bertrand competition only compassion between firms has an impact on equilibrium outcomes. Envy between firms has effect on equilibrium outcomes of Bertrand competition since the lowest equilibrium price, in the absence of inequity aversion, is equal to marginal cost. Additionally, the paper shows that only under very restrictive assumptions on preferences will compassion raise prices under symmetric Bertrand competition. For example, when there is Bertrand competition between two firms and marginal costs are constant, only if both firms are willing to give up more than one dollar of their profit to raise the average profit of their opponents by a dollar, can there be an equilibrium where price is above marginal

[9] In this paper a firm feels compassion towards its rivals when the average material payoff of its rivals is smaller than a firm's own material payoff. Similarly, a firm fells envy towards its rivals when the average material payoff of its rivals is greater than a firm's own material payoff.

[10] A similar result has also been found in a different context. Demougin and Fluet (2003) show that in a rank order tournament the principal is better off when agents are envious than when they are compassionate.

cost.[11] This finding is consistent with previous work on the impact of non-selfish preferences on perfectly competitive markets. Since the outcome of Bertrand competition is closer to the perfectly competitive outcome than the outcome of Cournot competition it is not surprisingly that non-selfish preferences play a much smaller role in Bertrand competition than in Cournot competition.[12]

The reminder of the paper proceeds as follows. Section 2 describes the most important types of non-selfish preferences. Section 3 sets up the model. Section 4 studies the impact of preferences for reciprocity on Cournot competition. Section 5 studies the impact of inequity aversion on Cournot competition. Section 6 considers the impact of non-selfish preferences on Bertrand competition. Section 7 concludes the paper. All proofs are in the Appendix.

2 Non-Selfish Preferences

A large number of studies shows that many people are not motivated exclusively by selfish motives. A person that exhibits non-selfish preferences cares about the material payoff allocated to her and also about the material payoff or the intentions of others. Non-selfish preferences have been shown to explain a wide range of behavior across different games.

A particularly important type of non-selfish preferences is the preference for reciprocity.[13] An individual with this type of preferences likes money but also responds to actions that are perceived to be kind in a kind manner and to actions that are perceived to be mean in a harmful manner. A person with a preference for reciprocity cares about the intentions behind the actions of their opponents but is not bothered by unfair payoff distributions. Preferences for reciprocity were first modeled in the economics literature by Rabin (1993) in the context of static games.[14]

A second type of non-selfish preferences is inequity aversion. Inequity aversion theories assume that individuals are concerned about their own material payoff but also the consequences of their acts on payoff distributions. An inequity averse person cares about the distribution of payoffs but it does not care about the intentions that lead others to choose certain actions.[15] There are two main theories of inequity aversion: Fehr and Schmidt's (1999) and Bolton and Ockenfels (2000). According to Fehr and Schmidt's (1999) model a player cares about his own payoff and dislikes absolute payoff differences between his own payoff and the payoff of any other player.[16] According to Bolton and Ockenfels's

[11] We show that this result also extends to Bertrand competition between two firms with increasing marginal costs.

[12] Bertrand competition is more competitive than Cournot competition according to the following criteria: lower mark-up/output ratios, larger average output, and lower average price.

[13] Preferences for reciprocity are also called preferences for process or intentions based fairness.

[14] Dufwenberg and Kirchsteiger (1998) extended it to dynamic games.

[15] Inequity aversion is sometimes called preference for outcome based fairness.

[16] Neilson (2000) provides an axiomatic characterization of the Fehr and Schmidt (1999) model of inequity aversion.

(2000) "Theory of Equity, Reciprocity, and Competition" (henceforth ERC) a players is concerned with both his own payoff and his relative share of the total group payoff.[17]

A third type of non-selfish preferences is pure altruism. An altruistic person always values positively the payoff of the opponents. An altruistic person is willing to increase the payoff of his opponents at a personal cost to himself, irrespective of the payoff distribution and irrespective of the behavior of the opponents. Finally, research also shown that some people exhibit spiteful preferences. A spiteful person always values negatively the payoff of his opponents. A spiteful person is willing to decrease the payoff of his opponents at a personal cost to himself, irrespective of the payoff distribution and irrespective of the behavior of the opponents.

Segal and Sobel (1999) provide an axiomatic foundation for non-selfish preferences that can reflect preferences for reciprocity, inequity aversion, altruism as well spitefulness. They assume that in addition to conventional preferences over outcomes, players in a strategic environment also have preferences over strategy profiles. Their representation theorem shows that the payoff function of a firm with such preferences is of the form

$$U_i(O(s_i, s^*_{-i})) = u_i(O(s_i, s^*_{-i})) + \sum_{j \neq i} w_{ij}(s_i, s^*_{-i}) u_j(O(s_i, s^*_{-i})), \tag{1}$$

where s_i is the strategy of player i, s^*_{-i} is the strategy that the rest of the players are playing, u_i is the utility from outcomes of player i, u_j is the utility from outcomes of player $j \neq i$, and w_{ij} is a coefficient that measures the weight player i gives to player j's utility, which is a function of the entire strategy profile. Positive values of the coefficient w_{ij} mean that player i is willing to sacrifice his payoff from outcomes in order to increase the payoff of player j. Negative values mean that player i is willing to sacrifice his payoff from outcomes in order to lower player j's payoff. Since the coefficient w_{ij} depends on the strategy chosen by player j, there is scope to model reciprocity. The underlying preferences in (1) are defined over outcomes. If an outcome specifies a material payoff to each player, then it is permissible for u_i to depend on other players' material payoffs. Thus, this approach also generalizes the inequity aversion approach.

3 Reciprocity

Let $N = \{1, 2, \ldots, n\}$ denote the set of firms. Let price be determined according to the inverse demand function $P(Q)$, where $Q = \sum q_i$. We make the standard assumption that $P(Q)$ is strictly positive on some bounded interval $(0, \bar{Q})$ with $P(Q) = 0$ for $Q \geq \bar{Q}$. We also assume that $P(Q)$ is strictly decreasing in the interval for which $P(Q) > 0$. Firms have costs of production given by $C_i(q_i)$.

[17] According to ERC, a player would be equally happy if all players received the same payoff or if some were rich and some were poor as long as he received the average payoff, while according to Fehr and Schmidt (1999) he would clearly prefer that all players get the same.

Firms costs of production are assumed to be increasing. To incorporate preferences for reciprocity into the Cournot oligopoly model we use Segal and Sobel (1999) approach and assume that the payoff function of firm i is given by

$$U_i(q_i, Q_{-i}) = \pi_i(q_i, Q_{-i}) + w_i\left(Q_{-i}, Q_{-i}^F\right) \sum_{j \neq i} \pi_j(q_i, Q_{-i}),$$

where $\pi_i(q_i, Q_{-i})$ is the material payoff of firm i and $w_i\left(Q_{-i}, Q_{-i}^F\right)$ is the weight that firm i places on its rivals aggregate material payoff $\sum_{j \neq i} \pi_j(q_i, Q_{-i})$. Firm i's material payoff depends on firm i's output, q_i, and on the aggregate output of its rivals, Q_{-i}, such that

$$\pi_i(q_i, Q_{-i}) = R_i\left(q_i, Q_{-i}\right) - C_i(q_i),$$

where $R_i(q_i, Q_{-i}) = P\left(Q\right) q_i$ is the revenue of the firm. We assume that the weight firm i places on its rivals aggregate material payoff depends on firm i's perception of the fair aggregate output of its rivals, Q_{-i}^F, and on the aggregate output of firm i's rivals. Furthermore, we assume that

$$w_i(Q_{-i}, Q_{-i}^F) \begin{cases} > 0 \text{ if } Q_{-i} < Q_{-i}^F, \\ = 0 \text{ if } Q_{-i} = Q_{-i}^F \\ < 0 \text{ otherwise} \end{cases}, \tag{2}$$

that is, firm i places a positive weight on its rivals aggregate material payoff when its rivals produce less than Q_{-i}^F, firm i places no weight on its rivals aggregate material payoff when its rivals produce Q_{-i}^F, and firm i places a negative weight on its rivals aggregate material payoff when its rivals produce more than Q_{-i}^F. These conditions capture the fact that a firm with reciprocal preferences cares about the intentions of its rivals. The first condition expresses positive reciprocity. If firm i expects the aggregate output of its rivals to fall short of its own perception of the fair aggregate output of its rivals, then firm i is willing to sacrifice some of its material payoff to increase its rivals' material payoffs. The third condition expresses negative reciprocity. When firm i expects its rivals to produce more than firm i's perception of the fair aggregate output of its rivals, then firm i is willing to sacrifice some of its material payoff to reduce its rivals' material payoffs.[18]

We assume throughout that firms' preferences for reciprocity as well as perceptions of the fair aggregate of its rivals are common knowledge. The problem of firm i is to maximize its payoff function taking the quantities produced by its rivals as given, that is, firm i solves the following problem

$$\max_{q_i} U_i(q_i, Q_{-i}) = \pi_i(q_i, Q_{-i}) + w_i\left(Q_{-i}, Q_{-i}^F\right) \sum_{j \neq i} \pi_j(q_i, Q_{-i}).$$

[18] Weighting functions that satisfy condition (2) arise naturally. For example, $w_i(Q_{-i}, Q_{-i}^F) = \alpha(Q_{-i}^F - Q_{-i})$, $w_i(Q_{-i}, Q_{-i}^F) = \alpha(Q_{-i}^F - Q_{-i})^3$, or $w_i(Q_{-i}, Q_{-i}^F) = \alpha\left(\frac{Q_{-i}^F}{Q_{-i}} - 1\right)$, with $\alpha > 0$.

When firm i has preferences for reciprocity its best reply to Q_{-i} is given by

$$r_i^R(Q_{-i}) = \arg_{q_i} \max P\left(Q\right) q_i - C_i(q_i) + w_i(Q_{-i}, Q_{-i}^F) \sum_{j \neq i} \left[P\left(Q\right) q_j - C_j(q_j)\right]. \tag{3}$$

Let $q^F = (Q_{-1}^F, Q_{-2}^F, \ldots, Q_{-n}^F)$ denote the vector of firms' perceptions of the fair aggregate output of their rivals. Let the n-firm Cournot oligopoly with reciprocal firms be denoted by $\Gamma^R(U, w, q^F)$. To begin our analysis we need to guarantee existence of equilibrium of $\Gamma^R(U, w, q^F)$.

There are four types of existence results which may apply to the Cournot model. The first type of result uses the standard existence theorem due to Nash and shows that every n-firm Cournot oligopoly has a Nash equilibrium if each firm's payoff is quasiconcave in q_i.[19]

The second type of result, due to Bamon and Frayssé (1985) and Novshek (1985), shows that every n-firm Cournot oligopoly has a Nash equilibrium if each firm's payoff depends on other firms' outputs only via their sum and marginal revenue is a decreasing function of the aggregate output of all other firms.

The third type of result deals with cases in which the Cournot game is a supermodular game. Here there are two different types of results, one for $n = 2$ and another one for $n \geq 2$. Milgrom and Roberts (1990) show that if the natural order on of one of the firms' action sets is reversed, then the Cournot duopoly is a supermodular game.[20] Amir (1996) provides conditions under which the n-firm Cournot oligopoly is a log-supermodular game. However, under these conditions, best replies are increasing which is not considered to be the "normal" case in Cournot games.

Finally, Tarsky (1955), McManus (1962, 1964), and Roberts and Sonnenschein (1977), show that every n-firm symmetric Cournot oligopoly has a Nash equilibrium if cost functions are convex.

Our goal is not only to prove existence of equilibria for the Cournot game with reciprocal firms but also to state comparative static results. The assumptions required to state each of the four existence results imply different trade-offs between generality in existence versus generality in comparative static results. We decide to focus on the Cournot duopoly case and treat it as a supermodular game. However, to provide intuition for some of the results we will often use the n-firm smooth version of the Cournot oligopoly game with quasiconcave and differentiable payoff functions.

Our first result guarantees that the Cournot duopoly game with reciprocal firms is a supermodular game.

Lemma 1: *If $n = 2$ and U_i has decreasing differences in (q_i, Q_{-i}), then $\Gamma^R(U, w, q^F)$ is a supermodular game.*

The assumption that the payoff function has decreasing differences in (q_i, Q_{-i}) means that the marginal returns to increasing a firm's output are lower if the rivals produce a higher output. Note that if firms are selfish, then the requirement

[19] This existence result is quite restrictive. See Ch. 4 in Vives (2001).

[20] This argument breaks down when there are three or more firms.

that π_i has decreasing differences in (q_i, Q_{-i}) boils down to the assumption that the revenue of firm i has decreasing differences in (q_i, Q_{-i}). However, if firms have preferences for reciprocity, then the requirement that U_i has decreasing differences in (q_i, Q_{-i}) also implies that the weight that preferences for reciprocity have on firm i's payoff function can not be too large by comparison to the weight of material payoffs. To best way to illustrate this point is to refer to a smooth version of the n-firm Cournot oligopoly game with reciprocal firms.[21] In that game the condition that U_i has decreasing differences in (q_i, Q_{-i}) is equivalent to the requirement that $\partial^2 U_i / \partial q_i \partial Q_{-i} < 0$, that is

$$\frac{\partial^2 U_i}{\partial q_i \partial Q_{-i}} = P'(Q) + P''(Q)\, q_i + \partial \left\{ w_i \left(Q_{-i}, Q_{-i}^F \right) P'(Q)\, Q_{-i} \right\} / \partial Q_{-i} < 0.$$

This condition is satisfied if the decreasing marginal revenue property holds, that is, $P'(Q) + P''(Q)\, q_i < 0$, and if the impact of a change in rivals' output on firm i's marginal payoff from preferences for reciprocity is relatively small by comparison with its impact on marginal revenue, that is

$$\partial \left\{ w_i \left(Q_{-i}, Q_{-i}^F \right) P'(Q)\, Q_{-i} \right\} / \partial Q_{-i} < \left| P'(Q) + P''(Q)\, q_i \right| .$$

Thus, if preferences for reciprocity are very important relative to material payoffs, then quantities may be strategic complements over some output ranges and strategic substitutes over others. If that happens, then we can no longer use the theory of supermodular games to state general results that characterize the impact of reciprocity on Cournot competition. Lemma 1 rules out this possibility.

If $\Gamma^R(U, w, q^F)$ is a supermodular game, then it follows from Topkis (1979), that the equilibrium set is non-empty and has a smallest and a largest pure-strategy Cournot-Nash equilibrium.[22] The next result shows how the equilibrium set of $\Gamma^R(U, w, q^F)$ changes with a change in firms' perceptions of the fair output of their rivals.

Theorem 1 *If $n = 2$, $\Gamma^R(U, w, q^F)$ is a supermodular game, and U_i has decreasing differences in $\left(q_i, Q_{-i}^F\right)$, then the smallest and the largest Cournot-Nash equilibria of $\Gamma^R(U, w, q^F)$ are nonincreasing functions of q^F.*

This result tells us that if firms have preferences for reciprocity, quantities are strategic substitutes (the weight of preferences for reciprocity is not too excessive), and the marginal returns from increasing production are decreasing with firms' perceptions of the fair output of their rivals, then the higher are

[21] In the smooth version of the n-firm Cournot oligoply game $P(Q)$ is twice continuously differentiable with $P'(Q) < 0$ (in the interval for which $P(Q) > 0$) and that the decreasing marginal revenue property holds, that is, $P'(Q) + P''(Q)\, q_i \leq 0$. Firms costs of production are assumed to twice continuously differentiable with $C_i' \geq 0$. The function $w_i(Q_{-i}, Q_{-i}^F)$ is assumed to be differentiable in both arguments with $\partial w_i / \partial Q_{-i} < 0$ and $\partial w_i / \partial Q_{-i}^F > 0$.

[22] Thes assumption that U_i has decreasing differences in (q_i, Q_{-i}) guarantees that best replies are decreasing and this implies existence of equilibrium.

firms' perceptions of the fair output of their rivals the lower will be the set of Cournot-Nash equilibria.[23]

The intuition behind this result is straightforward. The assumption that the payoff function has decreasing differences in (q_i, Q^F_{-i}) means that the larger a reciprocal firm perceives the fair output of their rivals to be, the smaller are the marginal returns from increasing production.[24] Thus, an increase in Q^F_{-i} shifts the best reply of a reciprocal firm i towards the origin. In other words, the more firm i perceives the fair output of its rivals to be high, the more firm i is willing to produce a smaller output level for any output level of its rivals. If this happens for every firm, then the higher are firms' perceptions of the fair output of their rivals the lower will be the set of Cournot-Nash equilibria.

Theorem 1 is a comparative statics result that characterizes the impact that firms' perceptions of the fair output of their rivals have on equilibrium quantities of Cournot competition among reciprocal firms. We are also interested in comparing the outcome of Cournot competition among reciprocal firms to that of Cournot competition among selfish firms. To do that we will compare the equilibria of game Γ^S, the supermodular Cournot game with selfish firms, to the equilibria of Γ^R, the supermodular Cournot game with reciprocal firms. We will assume that these two games are identical in all respects (market demand, costs, and number of firms) with the exception of firms' preferences. However, allowing for multiple equilibria makes the comparison cumbersome. Thus, we will assume that the game Γ^S has decreasing differences in (q_i, Q_{-i}), and that the firms' best replies have a slope greater than -1.[25] It is a well known result that these conditions guarantee that Γ^S has a unique equilibrium. Lemma 2 provides conditions under which the game Γ^R also has a unique equilibrium.

Lemma 2: *If $n = 2$, $\Gamma^R(U, w, q^F)$ is a supermodular game, and the firms' best*

[23] Note that this result does not imply that all Nash equilibria of $\Gamma(U, w, q^F)$ are nonincreasing functions of q^F. In fact we may have that a Nash equilibrium in the interior of the set of Nash equilibria of $\Gamma(U, w, \bar{q}^F)$ may be higher than the correspondent Nash equilibrium in the interior of the set of Nash equilibria of $\Gamma(U, w, \hat{q}^F)$ with $\hat{q}^F$ higher than $\bar{q}^F$. Still, a decrease in equilibrium output can be justified by a coordination argument since the smallest Cournot-Nash equilibrium is the most preferred equilibrium for the firms whereas the largest equilibrium is the less preferred one.

[24] In the smooth version of the n-firm Cournot oligopoly game with reciprocal firms the condition that U_i has decreasing differences in (q_i, Q^F_{-i}) is equivalent to the requirement that $\partial^2 U_i / \partial q_i \partial Q^F_{-i} < 0$. In that game we have that $\partial^2 U_i / \partial q_i \partial Q^F_{-i} = \left(\partial w_i / \partial Q^F_{-i} \right) P'(Q) Q_{-i}$. Since $P'(Q) < 0$ and $Q_{-i} > 0$ the condition holds if $\partial w_i / \partial Q^F_{-i} > 0$.

[25] In the smooth version of the n-firm Cournot oligopoly game with selfish firms these assumptions are satisfied if the decreasing marginal revenue property holds, marginal cost is increasing, and $P'(Q) - C''_i(q_i) < 0$, $i = 1, \ldots, n$. Under these conditions the profit of firm i is strictly concave in q_i.This follows since $\partial^2 \pi_i / \partial q_i^2 = P'(Q) + P''(Q) q_i + P'(Q) - C''_i(q_i) < 0$. We also have that $\partial^2 \pi_i / \partial q_i \partial Q_{-i} = P'(Q) + P''(Q) q_i < 0$. It also follows that the best reply function of firm i is has its slope is in the interval $(-1, 0)$:

$$r'_i(Q_{-i}) = -\frac{\partial^2 \pi_i / \partial q_i \partial Q_{-i}}{\partial^2 \pi_i / \partial q_i^2} = -\frac{P'(Q) + P''(Q) q_i}{P'(Q) + P''(Q) q_i + P'(Q) - C''_i(q_i)}.$$

Theorem 2.8 in Vives (2001) shows that these conditions imply that the smooth version of the n-firm Cournot oligopoly game with selfish firms has a unique equilibrium.

replies have a slope greater than -1, *then there exists an unique equilibrium of* $\Gamma^R(U, w, q^F)$.

This result guarantees that the supermodular Cournot game with reciprocal firms has a unique equilibrium. The condition that drives the result is the assumption that best replies have a slope strictly between $(-1, 0)$.[26] We are now ready to state the first result that compares the outcome of Cournot competition with reciprocal firms to that of Cournot competition with selfish firms.

Proposition 1: *If* $n = 2$, $\Gamma^S(\pi)$ *is a supermodular game such that the firms' best replies have a slope greater than* -1, $\Gamma^R(U, w, q^F)$ *is a supermodular game such that*
(i) $U_i = \pi_i + w_i \sum_{j \neq i} \pi_j$,
(ii) U_i *has decreasing differences in* (q_i, Q_{-i}^F),
(iii) the firms' best replies have a slope greater than -1, *and*
(iv) $Q_{-i}^F = Q_{-i}^{NS}$ *for all* i,
then the Nash equilibrium of $\Gamma^S(\pi)$ *coincides with that of* $\Gamma^R(U, w, q^F)$.

Proposition 1 says that if reciprocal firms perceive the fair output of its rivals to be equal to the output that its rivals would produce in the equilibrium of game with selfish firms, then the Cournot-Nash equilibrium of the game with reciprocal firms coincides with the Cournot-Nash equilibrium of the game with selfish firms. In this case market output, market price, consumer welfare, and firms profits are the same with reciprocal firms or with selfish firms.[27]

[26] In the smooth version of the n-firm Cournot oligopoly game with reciprocal firms the slope of the best reply of firm i is given by $r_i'(Q_{-i}) = -\frac{\partial^2 U_i / \partial q_i \partial Q_{-i}}{\partial^2 U_i / \partial q_i^2}$, where

$$\frac{\partial^2 U_i}{\partial q_i^2} = P'(Q) + P''(Q)\, q_i + P'(Q) - C''(q_i) + w_i(Q_{-i}, Q_{-i}^F) P''(Q)\, Q_{-i},$$

and

$$\frac{\partial^2 U_i}{\partial q_i \partial Q_{-i}} = P'(Q) + P''(Q)\, q_i + \partial \left\{ w_i(Q_{-i}, Q_{-i}^F) P'(Q)\, Q_{-i} \right\} / \partial Q_{-i}.$$

The slope is strictly above -1 if

$$\left| \partial \left\{ w_i(Q_{-i}, Q_{-i}^F) P'(Q)\, Q_{-i} \right\} / \partial Q_{-i} \right| < \left| P'(Q) - C''(q_i) + w_i(Q_{-i}, Q_{-i}^F) P''(Q)\, Q_{-i} \right|.$$

This condition implies that the game has a unique equilibrium by Theorem 2.8 in Vives (2001).

[27] It is possible that $q^{NR} \neq q^{NS}$ but $Q^{NR} = Q^{NS}$, that is the equilibrium strategy profile of the Cournot game with reciprocal firms differs from the equilibrium strategy profile of the Cournot game with selfish firms but the total output of each game is the same. This can happen when some firms' perception of fair aggregate output of its rivals is greater than the expected aggregate output of its rivals in the Nash equilibrium of the game with selfish firms and other firms have the opposite perception. For example, consider a symmetry Cournot game with 2 firms, with $w_i\left(q_j, q_j^F\right) = \alpha(q_j^F - q_j)$, with $i \neq j = 1, 2$, and where firm 1 perceives that $\triangle q_2 = q_2^F - q_2^{NS} > 0$, firm 2 perceives that $\triangle q_1 = q_1^F - q_1^{NS} < 0$, and $\triangle q_2 = |\triangle q_1|$. In this case we have that $q_1^{NR} < q_1^{NS}$ and $q_2^{NR} > q_2^{NS}$ but $Q^{NR} = Q^{NS}$. In this case preferences for reciprocity change best replies, market shares, and profits but do not change total output, market price and consumer welfare.

To clarify the intuition Proposition 1 we refer to the smooth n-firm Cournot oligopoly game with reciprocal firms. In that game the best reply of firm i to Q_{-i} is implicitly defined by the first-order condition

$$\frac{\partial U_i}{\partial q_i} = P(Q) + P'(Q)\,q_i - C_i'(q_i) + w_i(Q_{-i}, Q_{-i}^F)P'(Q)\,Q_{-i} = 0. \tag{4}$$

It is straightforward to interpret this condition. The term $P(Q) + P'(Q)\,q_i$ represents marginal revenue and the term $C_i'(q_i)$ marginal cost. These two terms represent the impact that a change in q_i has on firm i's material payoff.[28] The novelty here is the term $w_i(Q_{-i}, Q_{-i}^F)P'(Q)\,Q_{-i}$. This term represents the impact that a change in q_i has on firm i's payoff from preferences for reciprocity.

It follows from (4) that the best reply of a reciprocal firm i intercepts the best reply of a selfish firm i at $Q_{-i} = Q_{-i}^F$. This happens because $Q_{-i} = Q_{-i}^F$ implies $w_i(Q_{-i}, Q_{-i}^F) = 0$ and (4) reduces to $MR_i = MC_i$. Thus, if marginal revenue equals marginal cost for every firm, then market output as well as market price are the same with reciprocal firms or with selfish firms.

Proposition 1 tells us that a critical condition for the Cournot-Nash equilibrium of the game with reciprocal firms to differ from the Cournot-Nash equilibrium of the game with selfish firms is that reciprocal firms' perceptions of the fair output of its rivals are different from the equilibrium output of its rivals of the game with selfish firms.

Next we state our second result about reciprocity and Cournot competition.

Proposition 2: *If $n = 2$, $\Gamma^S(\pi)$ is a supermodular game such that the firms' best replies have a slope greater than -1, $\Gamma^R(U, w, q^F)$ is a supermodular game such that*
(i) $U_i = \pi_i + w_i \sum_{j \neq i} \pi_j$,
(ii) U_i has decreasing differences in (q_i, Q_{-i}^F),
(iii) the firms' best replies have a slope greater than -1, and
(iv) $Q_{-i}^F > Q_{-i}^{NS}$ for all i,
then the Nash equilibrium of $\Gamma^S(\pi)$ is greater than that of $\Gamma^R(U, w, q^F)$.

Proposition 2 tells us that if reciprocal firms perceive the fair output of their rivals to be greater than the equilibrium output that the rivals would produce in a game with selfish firms, then firms will produce a smaller output in the game with reciprocal firms than in the game with selfish firms. This is the positive reciprocity equilibrium. In such an equilibrium market output is smaller than market output in the equilibrium of the Cournot game with selfish firms. Thus, consumers are worse off if reciprocal firms' perceptions of fairness lead to a positive reciprocity equilibrium than if firms were selfish.

[28] In the smooth n-firm Cournot oligopoly game with selfish firms the best reply of firm i to Q_{-i} is the unique solution to the first-order condition

$$\frac{\partial \pi_i}{\partial q_i} = P(Q) + P'(Q)\,q_i - C_i'(q_i) = 0.$$

It follows from (4) that if firm i expects its rivals to produce an equilibrium output smaller than Q_{-i}^{F}, then firm i's best reply is to produce a smaller amount than the one it would produce if it was selfish. This happens because if $Q_{-i} < Q_{-i}^{F}$, then firm i places a positive weight on its rivals material payoff and this implies that $w_i(Q_{-i}, Q_{-i}^{F})P'(Q)\,Q_{-i} < 0$. In this case, if firm i produces less than its selfish best reply to Q_{-i} it has a first-order gain in payoff from positive reciprocity (it increases the material payoff of its opponents) and a second-order loss in material payoff (it reduces its own material payoff). Firm i will reduce production until the difference between marginal revenue and marginal cost equals the marginal payoff from positive reciprocity.

Next we state the "dual" result of Proposition 2.

Proposition 3: *If $n = 2$, $\Gamma^{S}(\pi)$ is a supermodular game such that the firms' best replies have a slope greater than -1, $\Gamma^{R}(U, w, q^{F})$ is a supermodular game such that*
(i) $U_i = \pi_i + w_i \sum_{j \neq i} \pi_j$,
(ii) U_i has decreasing differences in (q_i, Q_{-i}^{F}),
(iii) the firms' best replies have a slope greater than -1, and
(iv) $Q_{-i}^{F} < Q_{-i}^{NS}$ for all i,
then the Nash equilibrium of $\Gamma^{S}(\pi)$ is smaller than that of $\Gamma^{R}(U, w, q^{F})$.

This result tells us that if reciprocal firms perceive the fair output of their rivals to be smaller than the equilibrium output that the rivals would produce in a game with selfish firms, then firms will produce a larger output in the game with reciprocal firms than in the game with selfish firms. This is the negative reciprocity equilibrium. In such an equilibrium market output is larger than market output in the equilibrium of the Cournot game with selfish firms. Thus, consumers are better off if reciprocal firms' perceptions of fairness lead to a negative reciprocity equilibrium than if firms were selfish.

The intuition behind Proposition 3 can also be illustrated by (4). If a reciprocal firm i expects its rivals to produce an equilibrium output greater than Q_{-i}^{F}, then firm i's best reply is to produce a larger amount than the one it would produce if it was selfish. This happens because if $Q_{-i} > Q_{-i}^{F}$, then firm i places a negative weight on its rivals material payoff, that is, $w_i(Q_{-i}, Q_{-i}^{F}) < 0$. This in turn implies that $w_i(Q_{-i}, Q_{-i}^{F})P'(Q)\,Q_{-i} > 0$ since $P'(Q) < 0$ and > 0. If this is the case, then (4) is not satisfied if firm i would produce its selfish best reply to Q_{-i} since then we would have $MR_i - MC_i = 0$ but $w_i(Q_{-i}, Q_{-i}^{F})P'(Q)\,Q_{-i} > 0$. In fact, if firm i produces slightly more than its selfish best reply to Q_{-i} it has a first-order gain in payoff from negative reciprocity (it reduces the material payoff of its opponents) and a second-order loss in material payoff (it reduces its own material payoff). We see that (4) implies that firm i will increase production until the difference between marginal revenue and marginal cost equals the marginal payoff from negative reciprocity.

4 Inequity Aversion

Another important type of non-selfish preferences is inequity aversion. To study the impact of inequity aversion on Cournot competition we assume that firm i's payoff function is additively separable in the firm's own material payoff and the payoff of its rivals, that is

$$U_i(\pi_i, \pi_{-i}) = \pi_i + \sum_{j \neq i} \lambda_{ij}(\pi_j - \pi_i),$$

where λ_{ij} is a function that measures how differences in material payoffs between firm j and firm i have an impact on the weight that firm i puts on firm j's material payoff.[29] Furthermore, we assume that

$$\lambda_{ij}(\pi_j - \pi_i) \begin{cases} > 0 \text{ if } \pi_j < \pi_i \\ = 0 \text{ if } \pi_j = \pi_i \\ < 0 \text{ otherwise} \end{cases}, \tag{5}$$

that is, firm i places a positive weight on firm j's material payoff when j's material payoff is smaller than that of firm i, firm i places no weight on j's material payoff when firm j's material payoff equals that of firm i, and firm i places a negative weight on firm j's material payoff when j's material payoff is greater than that of firm i.

These conditions capture the fact that an inequity averse firm cares about the distribution of payoffs. The first condition expresses advantageous inequity or compassion. If firm i's material payoff is greater than that of firm j then firm i is willing to sacrifice some of its own material payoff to increase firm j's material payoff. The last third condition expresses disadvantageous inequity or envy. If firm i's material payoff is smaller than that of firm j then firm i is willing to sacrifice some of its own material payoff to reduce firm j's material payoff.

The problem of firm i is to maximize its payoff function taking the quantities produced by the other firms as given and taking into consideration the impact of its output on the distribution of material payoffs, that is

$$\max_{q_i} U_i(q_i, Q_{-i}) = \pi_i(q_i, Q_{-i}) + \sum_{j \neq i} \lambda_{ij}(\pi_j(q_i, Q_{-i}) - \pi_i(q_i, Q_{-i})).$$

The best reply of an inequity averse firm i to Q_{-i} is given by

$$r_i(Q_{-i}) = \arg_{q_i} \max \pi_i(q_i, Q_{-i}) + \sum_{j \neq i} \lambda_{ij}(\pi_j(q_i, Q_{-i}) - \pi_i(q_i, Q_{-i})).$$

We assume that the game is smooth and symmetric. Furthermore, we start our analysis by assuming that λ_{ij} is twice differentiable. If that is the case, then we can write the first-order condition to firm i's optimization problem as

$$\frac{\partial U_i}{\partial q_i} = \frac{\partial \pi_i}{\partial q_i} + \sum_{j \neq i} \lambda'_{ij}(\pi_j - \pi_i)\left(\frac{\partial \pi_j}{\partial q_i} - \frac{\partial \pi_i}{\partial q_i}\right) = 0, \tag{6}$$

[29] Neilson (2006) offers a full axiomatic characterization of this payoff function.

where $\partial\pi_i/\partial q_i = P(Q) + P'(Q)\, q_i - C_i'(q_i)$ and $\partial\pi_j/\partial q_i = P'(Q)\, q_j$, for all $j \neq i$. To guarantee that the first-order condition is the solution to firm i's the problem we will assume that the payoff function is strictly concave in q_i.

Lemma 3: *If*

$$\left|\sum_{j\neq i} \lambda_{ij}''(\pi_j - \pi_i)\left(\frac{\partial \pi_j}{\partial q_i} - \frac{\partial \pi_i}{\partial q_i}\right)^2 + \sum_{j\neq i} \lambda_{ij}'(\pi_j - \pi_i)\left(\frac{\partial^2 \pi_j}{\partial q_i^2} - \frac{\partial^2 \pi_i}{\partial q_i^2}\right)\right| \leq \left|\frac{\partial^2 \pi_i}{\partial q_i^2}\right|, \tag{7}$$

then there exists an equilibrium of the symmetric n-firm symmetric Cournot game with inequity averse firms and the equilibrium is unique.

Condition (7) guarantees that the payoff function of firm i is strictly concave in q_i. This guarantees existence of equilibrium. The assumption that the game is symmetric together with condition (7) imply that the equilibrium is unique. Let $q^{NI} = (q_1^{NI}, \ldots, q_n^{NI})$ denote the Nash equilibrium strategy profile of the n-firm Cournot game with inequity averse firms. We can now state the following result.

Proposition 4: *In the n-firm smooth and symmetric Cournot game with inequity averse firms if*
(i) $\lambda_{ij}(\pi_j - \pi_i)$ satisfies (5) and $\lambda_{ij}'(0) = 0$ for all i and j, then $q^{NI} = q^{NS}$;
(ii) $\lambda_{ij}(\pi_j - \pi_i)$ satisfies (5) and $\sum_{j\neq i} \lambda_{ij}'(0) < 0$ for all i, then $q^{NI} > q^{NS}$.

This result provides conditions under which differentiable specifications of inequity aversion will or will not change the equilibrium outcome of smooth and symmetric Cournot games. The result shows the main determinant of the impact of inequity aversion on these type of games is the sign of the first derivative of the weighting function evaluated at the point where material payoffs are equal.[30] If $\lambda_{ij}'(0) = 0$, then the market output with inequity averse firms is equal to the market output with selfish firms. By contrast, if $\sum_{j\neq i} \lambda_{ij}'(0) < 0$ for all i, then the market output with inequity averse firms is strictly greater than the market output with selfish firms.[31]

Proposition 4(i) shows that the equilibrium strategy profile of the symmetric Cournot game with inequity averse firms coincides with the equilibrium strategy profile of the symmetric Cournot game with selfish firms if the weighting function

[30] Assumption (5) and differentiability imply that $\lambda_{ij}'(0) \leq 0$, that is, it can not be that $\lambda_{ij}'(0) > 0$.

[31] It is hard to state general results that characterize the impact of inequity aversion on Cournot competition for asymmetric games. In asymmetric games firms may have different costs of production or different weight functions. If firms have different costs, then the most efficient firms will produce more output and have higher profits and the less efficient firms will produce less output and have lower profits. This implies that the most efficient firms will fell compassion toward the less efficient firms and the less efficient firms will feel envy toward the most efficient firms. This may lead the most efficient firms to produce less than in a game with selfish firms and the less efficient firms to produce more. Thus, it is not clear how aggregate output will change in an asymmetric Cournot oligopoly game when firms have aversion to inequality in payoffs.

satisfies condition (5) and $\lambda'_{ij}(0) = 0$ for all i and j.[32] The intuition behind this result is as follows. The fact that the game is symmetric together with the assumption that $\lambda'_{ij}(0) = 0$, for all i and j, imply inequity aversion pivots the best reply of each firm around the Nash equilibrium output of the Cournot game with selfish firms.[33] This type of inequity aversion changes the best reply functions of firms but does not change the equilibrium outcome of Cournot competition.

Proposition 4(ii) shows that the equilibrium strategy profile of the symmetric Cournot game with inequity averse firms is greater than the equilibrium strategy profile of the symmetric Cournot game with selfish firms if the weighting function satisfies condition (5) and $\sum_{j\neq i} \lambda'_{ij}(0) < 0$ for all i.[34] The fact that the game is symmetric together with the assumption that $\sum_{j\neq i} \lambda'_{ij}(0) < 0$ for all i imply inequity aversion pivots the best reply of firm i pivots around the point $q^p \in r_i^S(Q_{-i})$ such that $\sum_{j\neq i} \lambda'_{ij}(\pi_j - \pi_i)\left(q_j^p - q_i^p\right) < 0$.[35] This type of inequity aversion changes both the best reply functions of firms as well as the equilibrium outcome of Cournot competition.

Bolton and Ockenfels's (2000) were the first to study the impact of inequity aversion on equilibrium outcomes in oligopolistic markets. Bolton and Ockenfels find that differentiable inequity aversion has no implications in terms of equi-

[32] For example, the weighting function

$$\lambda_{ij}(\pi_j - \pi_i) = \begin{cases} -\alpha_{ij}(\pi_j - \pi_i)^2, & \text{if } \pi_j \geq \pi_i \\ \alpha_{ij}(\pi_j - \pi_i)^2, & \text{otherwise} \end{cases},$$

with $\alpha_{ij} > 0$, satisfies condition (5) $\lambda'_{ij}(0) = 0$ for all i and j.

[33] The way inequity aversion pivots best replies is identical to the way preferences for reciprocity pivot best replies. Consider, without loss of generality, Cournot competition between two firms. Suppose that firm 1 knows that firm 2 will produce a low output level (by comparison with the Nash equilibrium output level of the game with selfish firms). If firm 1 feels no compassion towards firm 2, then firm 1's best reply is to produce $r_1^S(q_2)$. However, if firm 1 feels compassion towards firm 2, then producing $r_1^S(q_2)$ is no longer the optimal choice. By producing somewhat less than $r_1^S(q_2)$ there is a second order loss in material payoff for firm 1 but a first order gain in reduction of advantageous inequity. By contrast, if firm 1 knows that firm 2 will produce a high output level and firm 1 feels no envy towards firm 2, then firm 1's best reply is to produce $r_1^S(q_2)$. However, if firm 1 feels envy towards firm 2, then producing $r_1^S(q_2)$ is no longer the optimal choice. By producing somewhat more than $r_1^S(q_2)$ there is a second order loss in material payoff for firm 1 but a first order gain in reduction of disadvantageous inequity.

[34] The weighting function

$$\lambda_{ij}(\pi_j - \pi_i) = -\alpha_{ij}\left[(\pi_j - \pi_i)^3 + (\pi_j - \pi_i)\right],$$

with $\alpha_{ij} > 0$, satisfies condition (5) and $\lambda'_{ij}(0) = -\alpha_{ij} < 0$.

[35] From (6) we see that the best reply of firm i pivots around the point $q^p = (q_i^p, Q_{-i}^p) \in r_i^S(Q_{-i})$ such that $\sum\limits_{j\neq i} \lambda'_{ij}(\pi_j - \pi_i)\left(\frac{\partial \pi_j}{\partial q_i} - \frac{\partial \pi_i}{\partial q_i}\right)$. This equality is equivalent to

$$-\sum_{j\neq i} \lambda'_{ij}(\pi_j - \pi_i)\left[P(Q) - C'_i(q_i)\right] = P'(Q)\sum_{j\neq i} \lambda'_{ij}(\pi_j - \pi_i)(q_j - q_i),$$

with $\lambda'_{ij}(\pi_j - \pi_i) < 0$ for all π_j and π_i. Since $P(Q) - C'_i(q_i) > 0$ and $P'(Q) < 0$ the equality is satisfied if $\sum\limits_{j\neq i} \lambda'_{ij}(\pi_j - \pi_i)(q_j - q_i) < 0$.

librium outcomes in Cournot and Bertrand competition. According to Bolton and Ockenfels's specification of inequity aversion the payoff function of a firm takes the form

$$U_i(\pi) = v\left(\pi_i, \frac{\pi_i}{\sum_{j=1}^{n} \pi_j}\right),$$

where the function v is assumed to be globally non-decreasing and concave in the first argument, to be strictly concave in the second argument (relative payoff), and to satisfy $v_2(\pi_i, 1/n) = 0$ for all π_i. Bolton and Ockenfels shows that this type of inequity aversion has no impact on equilibrium outcomes in symmetric Cournot games. Proposition 4 shows that Bolton and Ockenfels's result is driven by the assumption that $v_2(\pi_i, 1/n) = 0$ for all π_i.

We will now study the impact that Fehr and Schmidt's (1999) specification of inequity aversion has on equilibrium outcomes of Cournot competition.[36] We will show that this form of inequity aversion can change the strategic interaction between firms' choice variables. Recall that under the assumptions made in this paper the n-firm smooth Cournot game with selfish firms has best reply functions with a negative slope. This means that quantities are strategic substitutes. We will show that Fehr and Schmidt's (1999) specification of inequity aversion makes quantities become strategic complements over intermediate output levels. We will also show that if players with this type of preferences play Cournot games, then there can be a continuum of symmetric equilibria.

According to Fehr and Schmidt's (1999) specification firm i's payoff function is given by

$$U_i(\pi_i, \pi_{-i}) = \pi_i - \left[\frac{\alpha_i}{n-1}\sum_{j\neq i} \max(\pi_j - \pi_i, 0) + \frac{\beta_i}{n-1} \max \sum_{j\neq i} (\pi_i - \pi_j, 0)\right]. \tag{8}$$

The terms in the square bracket are the payoff effects of disadvantageous and advantageous inequity, respectively. We see that if firm i's material payoff is greater than the average material payoff of its rivals then firm i feels compassion towards its rivals, this is the advantageous inequity term. However, if firm i's material payoff is smaller than the average material payoff of its rivals then firm i feels envy towards its rivals, this is the disadvantageous inequity term.[37] We see that Fehr and Schmidt's model of inequity aversion has piecewise linear indifference curves over a firm's own material payoff and its rivals' material payoffs.

[36] This functional form of inequity aversion is widely used by the literature that studies the impact of inequity aversion on economic models.

[37] When there are only two firms in the market firm i's payoff becomes

$$U_i(\pi_i, \pi_j) = \pi_i - [\alpha_i \max(\pi_j - \pi_i, 0) + \beta_i \max(\pi_i - \pi_j, 0)], \ i \neq j = 1, 2. \tag{9}$$

Fehr and Schmidt assume that the dislike of disadvantageous inequity is stronger than that of advantageous inequity, i.e. $\alpha_i > \beta_i$ and that β_i is smaller than 1. We make no assumptions about the relation between α_i and β_i but we assume, like Fehr and Schmidt, that β_i is smaller than 1.

Thus, firm i's degree of inequity aversion towards its rivals is characterized by the pair of parameters (α_i, β_i), $i = 1, 2, \ldots, n$.[38] We say that firm i exhibits strict inequality aversion when both α_i and β_i are strictly greater than zero. We say that a firm is not averse to inequity when $\alpha_i = \beta_i = 0$. In all other cases we say that a firm is (weakly) averse to inequity. We assume that α_i and β_i, $i = 1, \ldots, n$, are common knowledge. We let the vector β denote firms' compassion degrees and we let the vector α denote firms' degrees of envy. Our next result characterizes a firm's best response in the presence of piecewise linear inequity aversion.

Proposition 5: *The best reply of firm i, $i = 1, \ldots, n$, in the n-firm Cournot game with piecewise linear inequity aversion is defined by*

$$r_i(Q_{-i}) = \begin{cases} s_i(Q_{-i}), & 0 \leq \frac{1}{n-1} Q_{-i} \leq q(\beta_i) \\ \frac{1}{n-1} Q_{-i}, & q(\beta_i) \leq \frac{1}{n-1} Q_{-i} \leq q(\alpha_i) \\ t_i(Q_{-i}), & q(\alpha_i) \leq \frac{1}{n-1} Q_{-i} \end{cases},$$

where

$$s_i(Q_{-i}) = \arg_{q_i} \max (1 - \beta_i) \left[P(Q) q_i - C_i(q_i) \right] + \frac{\beta_i}{n-1} \sum_{j \neq i} \left[P(Q) q_j - C_j(q_j) \right],$$

$$t_i(Q_{-i}) = \arg_{q_i} \max (1 + \alpha_i) \left[P(Q) q_i - C_i(q_i) \right] - \frac{\alpha_i}{n-1} \sum_{j \neq i} \left[P(Q) q_j - C_j(q_j) \right],$$

$q(\beta_i)$ is the solution to

$$(1 - \beta_i) \left[P(nq) - C_i'(q) \right] + P'(nq) q = 0, \tag{10}$$

and $q(\alpha_i)$ is the solution to

$$(1 + \alpha_i) \left[P(nq) - C_i'(q) \right] + P'(nq) q = 0. \tag{11}$$

Proposition 5 characterizes the impact of piecewise linear inequity aversion on a firm's optimal output choice for any output levels of its rivals. It tells us that a firm's best reply is continuous like in the standard Cournot game. However, by contrast with the standard Cournot game, a firm's best reply function in the Cournot game with piecewise linear inequity aversion is no longer monotonic.

[38] Alternatively, we could have considered that firm i has different feelings of compassion and envy towards each competitor. In this case we would have two inequity aversion parameters for each competitor per firm, that is, we would have α_{ij} and β_{ij} for $i \neq j = 1, \ldots, n$. We assume , like Ferh and Schmidt that firm i feels the same degree of envy and compassion towards all competitors. This makes the analysis simpler.

With piecewise linear inequity aversion the best reply has three different segments. When a firm's rivals produce low output levels the best response of a firm that feels inequity aversion has a negative slope and consists of a smaller level of output than the output level that the firm would produce if she felt no inequity aversion. When a firm's rivals produce intermediate output levels the best response of a firm that feels inequity aversion has a positive slope and consists in producing the average output level of the rivals. Finally, when a firm's rivals produce high output levels the best response of a firm that feels inequity aversion has a negative slope and consists of a larger level of output that the output level that the firm would produce if she felt no inequity aversion.

We can now ready to characterize the set of Nash equilibria of the n-firm symmetric Cournot oligopoly game when firms are averse to inequity in the sense of Fehr and Schmidt (1999). We do that in the next two results.

Proposition 6: *The unique Nash equilibrium of the n-firm symmetric Cournot game with selfish firms is always an equilibrium of the n-firm symmetric Cournot game with piecewise linear inequity averse firms.*

Recall that, under the assumptions made, there is a unique equilibrium of the symmetric Cournot game with selfish firms. In that equilibrium firms produce the same amount and the market price is between the perfectly competitive market price and the monopoly price. Proposition 6 shows that the unique Nash equilibrium of the Cournot game with selfish firms always belongs to the set of equilibria of the Cournot game with piecewise linear inequity aversion.

Proposition 7: *The set of Nash equilibria of the n-firm symmetric Cournot game with piecewise linear inequity averse firms is given by*

$$N^{IA} = \left\{(q_1, \ldots, q_n) : q_i = q_j,\ \forall i \neq j,\ \text{and } q(\beta) \leq q_i \leq q(\alpha),\ i = 1, \ldots, n\right\}, \tag{12}$$

where

$$q(\beta) = \max\left[q\left(\beta_1\right), \ldots, q(\beta_n)\right],$$

and

$$q(\alpha) = \min\left[q(\alpha_1), \ldots, q(\alpha_n)\right].$$

Proposition 7 tells us that if all firms are strictly averse to inequity, then there is a continuum of equilibria in the n-firm symmetric Cournot game with inequity averse firms. The intuition behind this result is straightforward. Consider, without loss of generality, Cournot competition between two firms. If a firm knows that the rival will produce the selfish Cournot-Nash quantity and the firm is averse to inequity, then its best reply is to produce also the selfish Cournot-Nash quantity. Producing an output different from the selfish Cournot-Nash quantity reduces the firm's material payoff and increases inequity costs.

Now, suppose that a firm knows that the rival will produce somewhat less than the selfish Cournot-Nash quantity. If the firm is averse to advantageous inequity, then its best reply is to produce exactly the same quantity as the rival. Producing a higher quantity than the rival increases the firm's material payoff by less than the cost from advantageous inequity. Similarly, if a firm knows that the rival will produce somewhat more than the selfish Cournot-Nash quantity, then its best reply is also to produce the same quantity as the rival. Producing a lower quantity than the rival increases the firm's material payoff by less than the cost from disadvantageous inequity.[39]

It follows that in some of the equilibria of the Cournot game with piecewise linear inequity aversion, the market price is lower than the equilibrium market price in the standard Cournot game whereas in other equilibria the market price is higher. Thus, it is not clear whether piecewise linear inequity aversion between firms is generally good or bad for consumers (or for firms).[40] However, we can state conditions under which piecewise linear inequity aversion is good or bad for consumers and firms. To do that we look at the impact of changes in the firms' degree of compassion and of envy.

Proposition 8: *The largest Nash equilibria of the n-firm symmetric Cournot game with piecewise linear inequity averse firms is a nondecreasing function of α. The smallest Nash equilibria is a nonincreasing function of β.*

This welfare result characterizes the impact of envy and compassion between firms on the set of Nash equilibria of the Cournot game with piecewise linear inequity averse firms. It tells us that there is a weak complementarity between the firms' degree of envy and their equilibrium output, that is, an increase in envy between firms increases the market output produced in the largest Nash equilibria of the Cournot model with inequity aversion. If that is the case, then an increase in the degree of envy between firms is likely to reduce firms' profits and increase consumer surplus.

On the other hand, Proposition 8 tells us that an increase in compassion between firms reduces the market output produced in the smallest Nash equilibria of the Cournot model with inequity aversion. If that is the case, then an increase in the degree of compassion between firms is likely to increase firms' profits and decrease consumer surplus. This result is quite intuitive. In fact, Fehr and Schmidt's payoff function implies that if firm i has a higher monetary payoff than the average payoff of her opponents and $\beta_i = 1/2$, then firm i is just as willing to keep one dollar to herself as to give it to her rivals. Now,

[39] The paper shows that a necessary condition for the continuum of equilibria to exist is that the game is not too asymmetric. In fact, when there are large cost asymmetries between firms the result no longer holds and there is a unique asymmetric equilibrium.

[40] Proposition 8 also shows that if there is at least one firm that is not averse to inequity, then there is a unique equilibrium of the symmetric Cournot game with piecewise linear inequity aversion: the equilibrium of the symmetric Cournot game with selfish firms. This point has been made before in papers that study the implications of interdependent preferences in ultimatum games and in papers that look at nonselfish preferences in perfectly competitive markets. See Bolton and Ockenfels (2000), Fehr and Schmidt (1999), and Segal and Sobel (2004).

suppose that all firms have the same preferences as firm i. In this case firms are acting as if they are maximizing their joint profit, $\sum \pi_i$. So, if $\beta_i = 1/2$, with $i = 1, \ldots, n$, then compassion leads to the collusive outcome.

The next result studies the implications of an increase in the number of firms when there is quantity competition in markets where firms have piecewise linear inequity aversion. To state this result we assume that α_i and β_i, $i = 1, \ldots, n$, are drawn from a uniform distribution with support on $[0,1]$.

Proposition 9: *As the number of firms increases the set of Nash equilibria of the n-firm symmetric Cournot game with piecewise linear inequity averse firms converges to the unique Nash equilibrium of n-firm symmetric Cournot game with selfish firms.*

This result shows that increasing the number of firms reduces the impact of piecewise linear inequity aversion on the set of Nash equilibria of the n-firm Cournot game. This happens because when there are n firms, the smallest Nash equilibria of the game is determined by the firm that has the lowest degree of compassion. Similarly, the largest Nash equilibria of the game is determined by the firm with the lowest degree of envy.[41] If the degree of compassion and of envy of each firm are drawn from a uniform distribution with support on $[0,1]$, then an increase in the number of firms makes it more likely that the lowest level of compassion is very close to zero and that the lowest level of envy is very close to zero. Thus, as the number of firms increases the smallest and the largest Nash equilibria of the n-firm symmetric Cournot game with inequity aversion converge to the Nash equilibrium of the standard n-firm symmetric Cournot game.[42]

5 Bertrand Competition

In the standard model of Bertrand competition firms select independently the price for the product and every firm has the commitment to supply whatever demand is forthcoming at the price it sets. Demand is strictly downward-sloping when positive, cutting both axes, and firms have increasing cost functions $C_i(q_i)$. Firms that set the lowest price split the demand and the remaining firms do not sell anything. That is, given a vector of prices $(p_i)_{i \in N}$ the sales of firm i are

$$q_i = \begin{cases} \frac{D(p_i)}{l}, & \text{if } p_j \geq p_i, \forall j \in N \\ 0, & \text{otherwise} \end{cases},$$

where $l = \#\{j \in N : p_j = p_i\}$.

It is a well know result that equilibrium outcomes of Bertrand competition depend on the shape of the cost function. Therefore, we will analyze the impact

[41] The same intuition is present in the first model in Fehr and Schmidt (1999).

[42] Huck et al. (2004) review of the evidence on experimental Cournot markets. They find that evidence from experimental Cournot games shows that when there are only two firms in the market collusive outcomes are frequent. However, as the number of firms increases output approaches the Nash-equilibrium.

of non-selfish preferences on Bertrand competition for different types of cost function.

5.1 Constant Marginal Costs

It is a well know result that if firms are selfish, marginal costs are constant and identical, then the only equilibrium is one where all firms set price equal to marginal cost, have zero profits, and split the market demand equally. Our next result characterizes the equilibrium of the n-firm symmetric Bertrand game when firms have piecewise linear inequity aversion and constant marginal costs.[43]

Proposition 10: *The set of Nash equilibria of the n-firm symmetric Bertrand game with piecewise linear inequity averse firms and constant marginal costs is given by*

$$p_i = \begin{cases} a, & \text{if } 1 - \frac{1}{n} \leq \min\left(\beta_1, \ldots, \beta_n\right) \\ c, & \text{otherwise} \end{cases},$$

where $a \in (c, \bar{p}]$, with $\bar{p}$ being the choke-off price for demand.

This result shows that if marginal cost are constant and there is at least one firm with a degree of compassion smaller than $1-1/n$, then the only equilibrium is for all firms to charge price equal to marginal cost. By contrast, if marginal costs are constant and all firms have a degree of compassion greater than $1-1/n$, then there is a continuum of symmetric equilibria where firms charge a price between marginal cost and the price that leads to zero market demand.

There are two interpretations for Proposition 10. For a fixed number of firms, this result tells us that piecewise linear inequity aversion can only raise price above marginal cost in Bertrand competition between firms with constant marginal costs when all firms have a very high level of compassion.[44] For a fixed level of compassion, say β, with $\beta \in (1/2, 1)$, this result tells us that an increase in the number of firms makes it is harder for piecewise linear inequity aversion to lead firms to set price above marginal cost. Of course, if we assume that β_i, $i = 1, \ldots, n$, has a uniform distribution on $[0, 1]$, then an increase in n raises $1 - \frac{1}{n}$ and reduces $\min\left(\beta_1, \ldots, \beta_n\right)$ which makes it even harder to satisfy the condition that allows firms to charge price above marginal cost.

5.2 Decreasing Returns

Dastidar (1995) shows that in symmetric Bertrand competition between firms with increasing marginal costs (decreasing returns), there is a continuum of symmetric equilibria where firms set a price in the interval $[p_L, p_H]$, and this interval contains the perfectly competitive price. To study Bertrand competition

[43] The findings with reciprocity and other forms of inequity aversion are similar.

[44] Recall that if $\beta = 1/2$ implies that a firm is just indifferent between keeping one dollar to heself and giving this dollar to her competitors.

with decreasing returns, n firms, and piecewise linear inequity aversion, we make two simplifying assumptions. We take demand to be linear–$D(p) = a - bp$–and we assume that firms' costs are given by $C_i(q_i) = cq_i^2/2$. If that is the case, then

$$p_L = \frac{ac}{2n + bc}, \tag{13}$$

and

$$p_H = \frac{ac}{2\left(1 - n^{-1}\right)\left(1 - n^{-2}\right)^{-1} + bc},$$

and the perfectly competitive price is given by $p = ac/(n+bc)$.[45] Our next result states the impact of piecewise linear inequity aversion on the n-firm symmetric Bertrand game with linear demand and quadratic costs.

Proposition 11: *The set of Nash equilibria of the n-firm symmetric Bertrand game with piecewise linear inequity averse firms, linear demand, and quadratic costs, is given by the price interval* $\left[p_L^{IA}, p_H^{IA}\right]$, *where* p_L^{IA} *is equal to (13) and*

$$p_H^{IA} = \min\left[p_H(\beta_1), \ldots, p_H(\beta_n)\right],$$

where

$$p_H(\beta_i) = \frac{ac}{2\left(1 - n^{-1} - \beta_i\right)\left(1 - n^{-2} - \beta_i\right)^{-1} + bc}, \; i = 1, \ldots, n, \tag{14}$$

and with $\beta_i \leq 1/2$, $i = 1, \ldots, n$.

Proposition 11 tells us that piecewise linear inequity aversion between firms may lead to a higher range of equilibrium prices when there is Bertrand competition and firms have increasing marginal costs. This happens because the upper bound of the equilibrium price interval in the Bertrand game with piecewise linear inequity aversion is greater than the upper bound of the equilibrium price interval in the Bertrand game with selfish firms whereas the lower bound stays the same.[46] Dastidar (1995) shows that the upper bound price is determined by a condition that makes a firm indifferent between playing the symmetric equilibrium and being the single producer in the market. The fact that a firm feels compassion for other firms reduces the payoff of being the single producer in the market and does not change a firm's profit when all firms are in the market. This implies that it is possible to have a larger range of higher prices in the symmetric equilibrium.

However, as we can see, the conditions for this to happen are quite restrictive. Like in the n-firm Cournot model with piecewise linear inequity aversion,

[45] See Vives (2001).

[46] Since β_i belongs to $[0, 1/2]$ for $i = 1, \ldots, n$, it follows that the denominator in (14) is smaller than the denominator in (13). If that is the case, then $p_H(\beta_i) > p_H$, for all i. If that is the case then $p_H^{IA} > p_H$, that is, the upper bound of the equilibrium price interval in the Bertrand game with inequity aversion is greater than the upper bound of the equilibrium price interval in the Bertrand game without inequity aversion.

increasing the number of firms in the n-firm Bertrand game with increasing marginal costs reduces the impact of piecewise linear inequity aversion on the set of equilibria.[47] Furthermore, like in the n-firm Cournot model, it is enough for one firm to be selfish for the effect to go away.

6 Conclusion

This paper studies the impact of non-selfish preferences among firms on quantity and price competition in oligopolistic markets. The paper focuses on reciprocity and inequity aversion since there are the two forms of non-selfish preferences that are more often found in experiments. The results obtained show that: (1) reciprocity and inequity aversion can be beneficial or harmful for consumers and firms, (2) reciprocity and inequity aversion have a larger impact on equilibrium outcomes of oligopolistic markets with a small number of firms, and (3) reciprocity and inequity aversion have a larger impact on equilibrium outcomes of Cournot competition than on equilibrium outcomes of Bertrand competition.

As it was mentioned in the introduction, this paper is a first step towards understanding the impact that non-selfish preferences have on oligopolistic markets. Our starting point was that firms only feel reciprocity or inequity aversion concerns among themselves. A natural extension of this paper would be to consider that firms' preferences also include consumers' welfare. It can also be interesting to find out what happens to market outcomes when some firms appeal to consumers' fairness concerns.[48] It is well known that in certain markets some firms donate a large share of their profits for charitable purposes while others do not.[49] It would be fruitful to explore the implications of these and related phenomena on equilibrium outcomes in oligopolistic markets.

[47] If one assumes that β_i is drawn from a uniform distribution on $[0, 1/2]$, then an increase in n makes it more likely that $\min(\beta_1, \ldots, \beta_n)$ is close to zero, which in turn implies that it is more likely that p_H^{IA} is close to p_H.

[48] Rabin (1993) considers a market where consumers perceive the price charged by a monopolist as unfair. He finds that the highest fairness equilibrium price is lower than the standard monopoly price.

[49] One of the most famous examples being Paul Newman's brand Newman's Own, Inc. which has given more than $175 million.dollars to thousands of charities since 1982.

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7 Appendix

Proof of Lemma 1: **[Incomplete]** The assumption that U_i has decreasing differences in (q_i, Q_{-i}) implies that all the conditions required for $\Gamma^R(U, w, q^F)$ to be a supermodular game are satisfied. *Q.E.D.*

Proof of Theorem 1: Theorem 6 in Milgrom and Roberts (1990). *Q.E.D.*

Proof of Lemma 2: Theorem 2.8 in Vives (2001). *Q.E.D.*

Proof of Proposition 1: We know by Lemma 2 that if the slope of the best replies is greater than -1, then there is a unique equilibrium of the supermodular Cournot game with reciprocal firms. Let $q^{NR} = (q_1^{NR}, \ldots, q_n^{NR})$ denote the Nash equilibrium strategy profile of the n-firm Cournot game with firms that have preferences for reciprocity. Let $q^{NS} = (q_1^{NS}, \ldots, q_n^{NS})$ denote the Nash equilibrium strategy profile of the n-firm Cournot game with selfish firms. We wish to show that if $Q_{-i}^F = Q_{-i}^{NS}$, then $q_i^{NR} = q_i^{NS}$, with $i = 1, \ldots, n,$. To do that we only need show that a reciprocal firm i has no incentive to deviate from $q_i^{NR} = q_i^{NS}$ when its rivals are playing $Q_{-i}^{NR} = Q_{-i}^{NS} = Q_{-i}^F$. If $Q_{-i}^F = Q_{-i}^{NS} = Q_{-i}^{NR}$ then $w_i(Q_{-i}, Q_{-i}^F) = 0$. If that is the case, the best response of firm i to Q_{-i}^{NR} is indeed $q_i^{NR} = q_i^{NS}$. *Q.E.D.*

Proof of Proposition 2: The assumptions made on π_i imply that $\Gamma^S(\pi)$ has a unique equilibrium. The assumptions made on U_i together with Lemma 2 imply that $\Gamma^R(U, w, q^F)$ also has a unique equilibrium. We know from Proposition 1 that if $\hat{q}^F = q^{NS}$, then $q^{NR} = q^{NS}$. If $\tilde{q}^F > \hat{q}^F = q^{NS}$, then Theorem 1 implies that the unique Cournot-Nash equilibrium of $\Gamma^R(U, w, \tilde{q}^F)$ is smaller than the unique Cournot-Nash equilibrium of $\Gamma^R(U, w, \hat{q}^F)$ *Q.E.D.*

Proof of Proposition 3: The assumptions made on π_i imply that $\Gamma^S(\pi)$ has a unique equilibrium. The assumptions made on U_i together with Lemma 2 imply that $\Gamma^R(U, w, q^F)$ also has a unique equilibrium. We know from Proposition 1 that if $\hat{q}^F = q^{NS}$, then $q^{NR} = q^{NS}$. If $\tilde{q}^F < \hat{q}^F = q^{NS}$, then Theorem 1 implies that the unique Cournot-Nash equilibrium of $\Gamma^R(U, w, \tilde{q}^F)$ is greater than the unique Cournot-Nash equilibrium of $\Gamma^R(U, w, \hat{q}^F)$. *Q.E.D.*

Proof of Lemma 3: To prove existence of equilibrium we only need to show that condition (**??**) implies that the payoff function of firm i is strictly concave in q_i. The second derivative of the payoff function of firm i is given by

$$\frac{\partial^2 U_i}{\partial q_i^2} = \frac{\partial^2 \pi_i}{\partial q_i^2} + \sum_{j \neq i} \lambda_{ij}''(\pi_j - \pi_i) \left(\frac{\partial \pi_j}{\partial q_i} - \frac{\partial \pi_i}{\partial q_i} \right)^2 + \sum_{j \neq i} \lambda_{ij}'(\pi_j - \pi_i) \left(\frac{\partial^2 \pi_j}{\partial q_i^2} - \frac{\partial^2 \pi_i}{\partial q_i^2} \right),$$

where $\partial^2\pi_i/\partial q_i^2 = 2P'(Q) + P''(Q)\, q_i - C''(q_i)$ and $\partial^2\pi_j/\partial q_i^2 = P''(Q)\, q_j$, for all $j \neq i$. Firm i's payoff function is strictly concave in q_i if $\partial^2 U_i/\partial q_i^2 < 0$. It is easy to check that condition (**??**) implies that $\partial^2 U_i/\partial q_i^2 < 0$. The assumption that the game is symmetric implies that there is a unique equilibrium. *Q.E.D.*

Proof of Proposition 4: The symmetry assumption implies that $q_i = q_j$, for all $i \neq j$, and this implies that $\pi_i = \pi_j$, for all $i \neq j$. If that is the case, then (6) is given by

$$\frac{\partial \pi_i}{\partial q_i} + \sum_{j\neq i} \lambda'_{ij}(0) \left(\frac{\partial \pi_j}{\partial q_i} - \left.\frac{\partial \pi_i}{\partial q_i}\right|_{q_i=q_j} \right) = 0,$$

with

$$\begin{aligned} \frac{\partial \pi_j}{\partial q_i} - \left.\frac{\partial \pi_i}{\partial q_i}\right|_{q_i=q_j} &= P'(Q)q_j - \left[P'(Q)q_i + P(Q) - C'(q_i)\right]\big|_{q_i=q_j} \\ &= -\left[P(Q) - C'(q_i)\right]. \end{aligned} \tag{15}$$

Using (15) the first-order condition becomes

$$\frac{\partial U_i}{\partial q_i} = P'(Q)q_i + \left[P(Q) - C'(q_i)\right] \left[1 - \sum_{j\neq i} \lambda'_{ij}(0)\right] = 0. \tag{16}$$

If $\lambda'_{ij}(0) = 0$ for all i and j, then (16) reduces to $\partial\pi_i/\partial q_i = 0$ and this implies $q^{NI} = q^{NS}$. This proves part (i). If $\sum_{j\neq i} \lambda'_{ij}(0) < 0$ for all i, then $1 - \sum_{j\neq i} \lambda'_{ij}(0) > 1$ in (16). Suppose, by contradiction that $q^{NI} = q^{NS}$. If that is the case, then $1 - \sum_{j\neq i} \lambda'_{ij}(0) > 1$ in (16) together with $P(Q) - C'(q_i) > 0$ and $P'(Q) < 0$ imply that the left-hand side of (16) is positive. Thus, $q^{NI} = q^{NS}$ is not an equilibrium when $\sum_{j\neq i} \lambda'_{ij}(0) < 0$ for all i. The assumption that marginal revenue is decreasing, that $C''_i(q_i) - P'(Q) > 0$ together with $\sum_{j\neq i} \lambda'_{ij}(0) < 0$ imply that $\partial^2 U_i/\partial q_i^2 < 0$. This in turn implies that $q^{NI} > q^{NS}$. This proves part (ii). *Q.E.D.*

Proof of Proposition 5: To prove this result we will start by showing that $q(\alpha_i)$ is an increasing function of α_i and that $q(\beta_i)$ is a decreasing function of β_i for $i = 1, \ldots, n$. From (11) we have

$$h(q, \alpha_i) = (1 + \alpha_i) \left[P(nq) - C'_i(q)\right] + P'(nq)q = 0,$$

which implies

$$\frac{\partial q}{\partial \alpha_i} = -\frac{\partial h/\partial \alpha_i}{\partial h/\partial q} = -\frac{P(Q) - C'_i(q)}{(1 + n(1 + \alpha_i))\, P'(Q) + nP''(Q)q - C''_i(q)} > 0,$$

since we have assumed that $P'(Q) < 0$, $P'(Q) \leq 0$, and $C_i''(q_i) \geq 0$. From (10) we have

$$g(q, \beta_i) = (1 - \beta_i) \left[P(nq) - C_i'(q) \right] + P'(nq)q = 0,$$

which implies

$$\frac{\partial q}{\partial \beta_i} = -\frac{\partial g / \partial \beta_i}{\partial g / \partial q} = -\frac{- \left[P(Q) - C_i'(q) \right]}{\left(1 + n(1 - \beta_i)\right) P'(Q) + nP''(Q)q - C_i''(q)} < 0,$$

since we have assumed that $P'(Q) < 0$, $P'(Q) \leq 0$, and $C_i''(q_i) \geq 0$.
We will now show that $q_i = \frac{1}{n-1} \sum_{j \neq i} q_j$ is a best response for firm i when the rivals produce

$$q_i^N \leq \bar{q}_j \leq q(\alpha_i), \tag{17}$$

where $\bar{q}_j = \frac{1}{n-1} \sum_{j \neq i} q_j$. To do that we will show that firm i can not gain from deviating from $q_i = \bar{q}_j$ when (17) holds. Suppose, that (17) holds and that firm i produces $q_i = \bar{q}_j + \varepsilon$, with $\varepsilon > 0$. In this case firm i's payoff is given by

$$U_i = (1 - \beta_i) \left[P\left(Q\right) q_i - C_i(q_i) \right] + \frac{\beta_i}{n-1} \sum_{j \neq i} \left[P\left(Q\right) q_j - C_j(q_j) \right]$$

and the change in firm i's payoff from producing $q_i = \bar{q}_j + \varepsilon$, $\varepsilon > 0$, instead of $\bar{q}_j$ is approximately equal to

$$\begin{aligned} dU_i &\approx \left. (1 - \beta_i) \left[P'\left(Q\right) q_i + P(Q) - C_i'(q_i) \right] + \frac{\beta_i}{n-1} \sum_{j \neq i} P'\left(Q\right) q_j \right|_{q_i = \bar{q}_j} (\varepsilon) \\ &= \left[\left(P'\left(n\bar{q}_j\right) \bar{q}_j + P(n\bar{q}_j) - C_i'(\bar{q}_j) \right) - \beta_i \left(P(n\bar{q}_j) - C_i'(\bar{q}_j) \right) \right] \varepsilon. \end{aligned}$$

The square brackets are negative since $q_i = \bar{q}_j > \arg\max \left[P\left(Q\right) q_i - C_i(q_i) \right]$ and $P(n\bar{q}_j) - C_i'(\bar{q}_j) > 0$. So, when (17) holds, firm i can not gain by producing more than $\bar{q}_j$. Now, suppose that (17) holds and that firm i produces $q_i = \bar{q}_j + \varepsilon$, with $\varepsilon < 0$. In this case firm i's payoff is given by

$$U_i = (1 + \alpha_i) \left[P\left(Q\right) q_i - C_i(q_i) \right] - \frac{\alpha_i}{n-1} \sum_{j \neq i} \left[P\left(Q\right) q_j - C_j(q_j) \right],$$

and the change in firm i's payoff from producing $q_i = \bar{q}_j + \varepsilon$, $\varepsilon < 0$, instead of $\bar{q}_j$ is approximately equal to

$$\begin{aligned} dU_i &\approx \left. (1 + \alpha_i) \left[P'\left(Q\right) q_i + P(Q) - C_i'(q_i) \right] - \frac{\alpha_i}{n-1} \sum_{j \neq i} P'\left(Q\right) q_j \right|_{q_i = \bar{q}_j} (\varepsilon) \\ &= \left[(1 + \alpha_i) \left[P(n\bar{q}_j) - C_i'(\bar{q}_j) \right] + P'(n\bar{q}_j)\bar{q}_j \right] \varepsilon \\ &= \left. h(q, \alpha_i) \right|_{q = \bar{q}_j} (\varepsilon). \end{aligned}$$

Since $\varepsilon < 0$, we have that $sign\ dU_i = -sign\ \left. h(q,\alpha_i)\right|_{q=\bar{q}_j}$. If $\bar{q}_j = q(\alpha_i)$ we have that $sign\ dU_i = 0$. If $q_i^N \leq \bar{q}_j < q(\alpha_i)$, the fact $h(q,\alpha_i)$ is a decreasing function of q implies that $\left. h(q,\alpha_i)\right|_{q=\bar{q}_j} > 0$, which in turn implies that $sign\ dU_i < 0$. So, when (17) holds, firm i can not gain by producing less than $\bar{q}_j$. From this result is follows immediately that if firm i's rivals produce $q(\alpha_i) < \frac{1}{n-1}\sum_{j\neq i} q_j$, then the best response of firm i is given by $t_i(q_{-i})$.
We will now show that $q_i = \frac{1}{n-1}\sum_{j\neq i} q_j$ is a best response for firm i when the rivals produce

$$q(\beta_i) \leq \bar{q}_j \leq q_i^N, \tag{18}$$

To do that we will show that firm i can not gain from deviating from $q_i = \bar{q}_j$ when (18) holds. Suppose, that (18) holds and that firm i produces $q_i = \bar{q}_j + \varepsilon$, with $\varepsilon < 0$. In this case firm i's payoff is given by

$$U_i = (1+\alpha_i)\left[P\left(Q\right)q_i - C_i(q_i)\right] - \frac{\alpha_i}{n-1}\sum_{j\neq i}\left[P\left(Q\right)q_j - C_j(q_j)\right],$$

and the change in firm i's payoff from producing $q_i = \bar{q}_j + \varepsilon$, $\varepsilon < 0$, instead of $\bar{q}_j$ is approximately equal to

$$\begin{aligned} dU_i &\approx \left.(1+\alpha_i)\left[P'\left(Q\right)q_i + P(Q) - C_i'(q_i)\right] - \frac{\alpha_i}{n-1}\sum_{j\neq i}P'\left(Q\right)q_j\right|_{q_i=\bar{q}_j} (\varepsilon) \\ &= \left[(1+\alpha_i)\left[P'(n\bar{q}_j)\bar{q}_j + P(n\bar{q}_j) - C_i'(\bar{q}_j)\right] - \alpha_i P'(n\bar{q}_j)\bar{q}_j\right]\varepsilon. \end{aligned}$$

The square brackets are positive since $q_i = \bar{q}_j < \arg\max\left[P\left(Q\right)q_i - C_i(q_i)\right]$ and $P'(n\bar{q}_j) < 0$. So, when (18) holds, firm i can not gain by producing less than $\bar{q}_j$. Now, suppose that (18) holds and that firm i produces $q_i = \bar{q}_j + \varepsilon$, with $\varepsilon > 0$. In this case firm i's payoff is given by

$$U_i = (1-\beta_i)\left[P\left(Q\right)q_i - C_i(q_i)\right] + \frac{\beta_i}{n-1}\sum_{j\neq i}\left[P\left(Q\right)q_j - C_j(q_j)\right]$$

and the change in firm i's payoff from producing $q_i = \bar{q}_j + \varepsilon$, $\varepsilon > 0$, instead of $\bar{q}_j$ is approximately equal to

$$\begin{aligned} dU_i &\approx \left.(1-\beta_i)\left[P'\left(Q\right)q_i + P(Q) - C_i'(q_i)\right] + \frac{\beta_i}{n-1}\sum_{j\neq i}P'\left(Q\right)q_j\right|_{q_i=\bar{q}_j} (\varepsilon) \\ &= \left[(1-\beta_i)\left[P(n\bar{q}_j) - C_i'(\bar{q}_j)\right] + P'\left(n\bar{q}_j\right)\bar{q}_j\right]\varepsilon \\ &= \left. g(q,\beta_i)\right|_{q=\bar{q}_j}(\varepsilon). \end{aligned}$$

Since $\varepsilon > 0$, we have that $sign\ dU_i = sign\ \left. g(q,\beta_i)\right|_{q=\bar{q}_j}$. If $\bar{q}_j = q(\beta_i)$ we have that $sign\ dU_i = 0$. If $q(\beta_i) < \bar{q}_j \leq q_i^N$, the fact $g(q,\beta_i)$ is a decreasing function of q implies that $\left. g(q,\beta_i)\right|_{q=\bar{q}_j} < 0$, which in turn implies that $sign\ dU_i < 0$. So, when (18) holds, firm i can not gain by producing more than $\bar{q}_j$. From this result is follows immediately that if firm i's rivals produce $0 \leq \frac{1}{n-1}\sum_{j\neq i} q_j < q(\beta_i)$, then the best response of firm i is given by $s_i(q_{-i})$. *Q.E.D.*

Proof of Proposition 6: We wish to show that $q_i = q_i^N$ is the best response to $q_{-i}^N = \left(q_1^N, \ldots, q_{i-1}^N, q_{i+1}^N, \ldots q_n^N\right)$ in the n-firm symmetric Cournot game with piecewise linear inequity averse firms. The welfare of firm 1 under outcome q^N is given by $\pi_1(q^N) = \left[P\left(nq_i^N\right) - C_i(q_i^N)\right] q_i^N$, where $q_i^N = \arg_{q_1}\max\left[P\left(q_i + \sum_{j\neq i} q_j^N\right) - C_i(q_i)\right] q_i$.
If firm i produces $q_i^N + \varepsilon$, with $\varepsilon > 0$, and all other firms produce q_{-i}^N, then the change in firm i's profit is approximately equal to

$$\begin{aligned} d\pi_i &\approx \varepsilon\, \partial\pi_i/\partial q_i\big|_{q_i=q_i^N} + \frac{1}{2}\varepsilon^2\, \partial^2\pi_i/\partial q_i^2\big|_{q_i=q_i^N} \\ &= \frac{1}{2}\varepsilon^2\left[2P'(Q^N) + P''(Q^N)q_i^N - C''(q_i^N)\right]. \end{aligned} \tag{19}$$

The assumption that $P' < 0$, $P'' \leq 0$, and $C'' \geq 0$ imply that $d\pi_i < 0$. The change in the profit in one of firm i's rivals, say firm j, is approximately equal to

$$\begin{aligned} d\pi_j &\approx \varepsilon\, \partial\pi_j/\partial q_i\big|_{q_i=q_i^N} + \frac{1}{2}\varepsilon^2\, \partial^2\pi_j/\partial q_i^2\big|_{q_i=q_i^N} \\ &= \varepsilon P'(Q^N)q_j^N + \frac{1}{2}\varepsilon^2 P''(Q^N)q_j^N. \end{aligned}$$

Note that the change in the average profit of firm i's rivals is the same as the change in the profit of a single competitor since

$$\begin{aligned} \frac{1}{n-1}\sum_{j\neq i} d\pi_j &\approx \frac{1}{n-1}\varepsilon P'(Q^N)\sum_{j\neq i} q_j^N + \frac{1}{2}\varepsilon^2 P''(Q^N)\sum_{j\neq i} q_j^N \\ &= \varepsilon P'(Q^N)q_j^N + \frac{1}{2}\varepsilon^2 P''(Q^N)q_j^N. \end{aligned} \tag{20}$$

The assumption that $P' < 0$ and $P'' \leq 0$ imply that $\frac{1}{n-1}\sum_{j\neq i} d\pi_j < 0$. We see from (19) and (20) that if firm i produces $q_i^N + \varepsilon$, with $\varepsilon > 0$, and all other firms produce q_{-i}^N, then there is a first order decrease in profits of firm i and a second order decrease in the average profit of firm i's rivals. Thus, if firm i produces $q_i^N + \varepsilon$, with $\varepsilon > 0$, it suffers a loss in profits and also a loss from an increase in inequity aversion given that the average profit of the rivals becomes smaller than firm i's profit. If that is the case, then firm i can not gain by producing $q_i^N + \varepsilon$, with $\varepsilon > 0$, instead of producing q_i^N.
If firm i produces $q_i^N + \varepsilon$, with $\varepsilon < 0$, and all other firms produce q_{-i}^N, then the change in firm i's profit is given by (19) and we have that $d\pi_i < 0$. The change in the average profit of firm i's rivals is given by (20) and we have that $\frac{1}{n-1}\sum_{j\neq i} d\pi_j > 0$ since $\varepsilon < 0$ and the first term is of first order while the second term is of second order. Thus, if firm i produces $q_i^N + \varepsilon$, with $\varepsilon < 0$,

it suffers a loss in profits and also a loss from an increase in inequity aversion given that the average profit of the rivals becomes greater than firm i's profit. If that is the case, then firm i can not gain by producing $q_i^N + \varepsilon$, with $\varepsilon < 0$, instead of producing q_i^N. This proves that $q_i = q_i^N$ is the best response to $q_{-i}^N = \left(q_1^N, \ldots, q_{i-1}^N, q_{i+1}^N, \ldots q_n^N\right)$ in the n-firm symmetric Cournot game with piecewise linear inequity averse firms. But, this in turn implies that q^N is a Nash equilibrium of the n-firm symmetric Cournot game with piecewise linear inequity averse firms. *Q.E.D.*

Proof of Proposition 7: We know that the set N^{IA} is non-empty since it contains at least the Nash equilibrium of the standard n-firm symmetric Cournot game. We will now show that if all firms display strict inequity aversion, then $q(\beta) < q(\alpha)$, that is, N^{IA} is an interval. We know that $q(\alpha_i)$ is an increasing function of α_i and that $q(\beta_i)$ is a decreasing function of β_i for $i = 1, \ldots, n$. It is obvious that if at least one firm does not feel inequity aversion then $q(\beta) = q(\alpha)$, and N^{IA} is a singleton. To see this suppose that firm i does not feel inequity aversion, that is, $\alpha_i = \beta_i = 0$. If this is the case, then (10) and (11) imply that $q(0) = q^N$. If $q(\alpha_i)$ is an increasing function of α_i and $q(0) = q^N$, then $q(\alpha) = q^N$. Similarly, if $q(\beta_i)$ is a decreasing function of β_i and $q(0) = q^N$, then $q(\beta) = q^N$. So, if at least one firm does feel inequity aversion we have that $q(\beta) = q(\alpha) = q^N = N^{IA}$. We will now show that if all firms display strict inequity aversion, then $q(\beta) < q(\alpha)$, that is, N^{IA} is an interval. If all firms display strict inequity aversion, $q(\alpha_i)$ is an increasing function of α_i and $q(0) = q^N$, then $q(\alpha) > q^N = q(0)$. Also, if all firms display strict inequity aversion, $q(\alpha_i)$ is an decreasing function of β_i and $q(0) = q^N$, then $q(\beta) < q^N = q(0)$. This shows that $q(\beta) < q(\alpha)$ when all firms display strict inequity aversion, that is the set N^{IA} is an interval. All outcomes in the set N^{IA} are equilibria of the symmetric Cournot game with inequity aversion since for any profile of quantities, q_{-i}, the quantity q_i belongs to the best response of firm i, $i = 1, \ldots n$. *Q.E.D.*

Proof of Proposition 8: The quantity produced by each firm in the largest Nash equilibria of N^{IA} is given by $q(\alpha) = \min\left[q(\alpha_1), \ldots, q(\alpha_n)\right]$. The largest Nash equilibria of N^{IA} is nondecreasing in α since $\min\left[q(\alpha_1), \ldots, q(\alpha_n)\right]$ is nondecreasing in α. Similarly, the quantity produced by each firm in the smallest Nash equilibria of N^{IA} is given by $q(\beta) = \max\left[q(\beta_1), \ldots, q(\beta_n)\right]$. The smallest Nash equilibria of N^{IA} is nonincreasing in β since $\max\left[q(\beta_1), \ldots, q(\beta_n)\right]$ is nonincreasing in β. *Q.E.D.*

Proof of Proposition 9: When all firms feel strict inequity aversion it follows that $q(\beta) < q^N < q(\alpha)$. Since α_i is drawn from a uniform distribution with support on $[0, 1]$, the larger is n the most likely it becomes that $\min\left(\alpha_1, \ldots \alpha_n\right)$ is closer to zero, this in turn implies that the larger is n the most likely is that $N(\alpha)$ is closer to q^N. Similarly, since β_i is drawn from a uniform distribution with support on $[0, 1]$, the larger is n the most likely it becomes that $\min\left(\beta_1, \ldots, \beta_n\right)$

is closer to zero, this in turn implies that the larger is n the most likely is that $N(\beta)$ is closer to q^N. *Q.E.D.*

Proof of Proposition 10: If marginal costs are constant, then we have $C_i(q_i) = cq_i,\ i = 1, \dots, n$. The payoff of firm i in the presence of piecewise linear inequity aversion is given by

$$U_i\left(p_i, p_{-i}\right) = \begin{cases} \left(1-\beta_i\right)\left(p_i - c\right)D(p_i), & \text{if } \ p_i < p_j^{\min} \\ \left(1-\beta_i+\beta_i\frac{l-1}{n-1}\right)\frac{(p_i-c)D(p_i)}{l}, & \text{if } \ p_j \geq p_i, \quad \forall j \in N \\ -\alpha_i\left(p_j^{\min}-c\right)D(p_j^{\min}), & \text{if } \ p_i > p_j^{\min} \end{cases},$$

where $p_j^{\min} = \min\left(p_1, \dots, p_{i-1}, p_{i+1}, \dots, p_n\right)$ and $l = \#\left\{j \in N : p_j = p_i\right\}$. For firm i not to deviate from an equilibrium where firm i plus $l-1$ firms charge a price $p \in (c, \bar{p}]$ and the remaining firms charge a higher price than p it must be that

$$\left(1-\beta_i\right)\left(p_i - c\right)D(p_i) \leq \left(1-\beta_i+\beta_i\frac{l-1}{n-1}\right)\frac{\left(p_i-c\right)D(p_i)}{l}$$

or

$$n-1 \leq \frac{\beta_i}{1-\beta_i}$$

or

$$1-\frac{1}{n} \leq \beta_i.$$

For all firms not to deviate, the case when $l = n$, from such an equilibrium we need that

$$1-\frac{1}{n} \leq \min\left(\beta_1, \dots, \beta_n\right).$$

If this condition does not hold, then there is at least one firm that is always willing to undercut a price $p \in (c, \bar{p}]$. If that is the case, then the only equilibrium is for all firms to charge price equal to marginal cost. *Q.E.D.*

Proof of Proposition 11: If all firms produce in the market and all charge the same price the payoff of each firm is given by $\pi_n(p) = pD(p)/n - C(D(p)/n)$. If demand is $D(p) = a - bp$ and $C(q) = cq^2/2$ we have that $\pi_n(p) = p(a - bp)/n - c(a-bp)^2/2n^2$. Dastidar (1995) shows that the lower bound of the set of equilibrium prices, p_L, is given by the solution to $\pi_n(p) = 0$. In this case, our assumptions imply that $p_L = ac/(2n+b)$. If a firm is the single producer in the market, then all the other firms must have zero profit and therefore the profit of the single producer is given by $\pi_1(p) = pD(p) - C(D(p))$. However, with inequity aversion, the payoff of this firm is given by $U_i(p) =$

$(1-\beta_i)\pi_1(p)$. If demand is $D(p)=a-bp$ and $C(q)=cq^2/2$ we have that $U_i(p)=(1-\beta_i)\left[p(a-bp)-c(a-bp)^2/2\right]$. Applying Dastidar (1995) to our model with inequity aversion, we have that the upper bound of the set of equilibrium prices with inequity aversion, p_H, is the given by (14), where $p_H(\beta_i)$ solution to $U_i(p)=\pi_n(p)$. That is, $p_H(\beta_i)$ is the solution to

$$(1-\beta_i)\left[p(a-bp)-c(a-bp)^2/2\right]=p(a-bp)/n-c(a-bp)^2/2n^2.$$

Solving this equation for p we obtain

$$\begin{aligned}
p_H(\beta_i) &= \frac{ac\left(n^2-1-\beta_i n^2\right)}{2(n^2-n-\beta_i n^2)+bc(n^2-1-\beta n^2)} \\
&= \frac{ac}{2\left(n^2-n-\beta_i n^2\right)\left(n^2-1-\beta_i n^2\right)^{-1}+bc} \\
&= \frac{ac}{2\left(1-n^{-1}-\beta_i\right)\left(1-n^{-2}-\beta_i\right)^{-1}+bc}.
\end{aligned}$$

Q.E.D.